AEDC-TDR-64-61

LOCAL HEAT-TRANSFER COEFFICIENTS IN A NOZZLE WITH HIGH-SPEED LAMINAR FLOW

By

William H. Carden
von Kármán Gas Dynamics Facility
ARO, Inc.

TECHNICAL DOCUMENTARY REPORT NO. AEDC-TDR-64-61

April 1964

Program Element 62405334/8950, Task 895004

(Prepared under Contract No. AF 40(600)-1000 by ARO, Inc., contract operator of AEDC, Arnold Air Force Station, Tenn.)

ARNOLD ENGINEERING DEVELOPMENT CENTER
AIR FORCE SYSTEMS COMMAND
UNITED STATES AIR FORCE

NOTICES

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ARO Project No. VL2407

FOREWORD

The author is indebted to his colleagues for their assistance during this investigation. Max Kinslow and J. C. Piggott were primarily responsible for the design of the experimental nozzle. R. F. Armstrong contributed his ingenuity in the fabrication of a special instrumentation system. W. H. Sims and W. C. Armstrong provided computer solutions for the nozzle flow properties. C. H. Lewis brought the similar solution of Beckwith and Cohen to the attention of the author. Finally, the author is particularly indebted to J. Leith Potter, who provided much assistance and advice in the analytic portion of this investigation.

ABSTRACT

An experimental investigation of the local heat-transfer coefficient in the throat region of a nozzle operating under conditions of cold wall, thick laminar boundary-layer, low-density, high-speed flow has been conducted. The experimental results have been compared with several analytic procedures for predicting the heat-transfer coefficient in laminar flow. The simple flat plate equation is shown to underestimate the heat-transfer coefficient, while the method of Cohen and Reshotko predicts coefficients which are too large. The incremental flat plate method of Pasqua and Stevens and a modification of a solution of Beckwith and Cohen exhibit good results downstream of the nozzle throat when referenced to the value calculated by the simple flat plate equation at the nozzle exit. The effects of thermal radiation upstream of the nozzle throat are indicated.

PUBLICATION REVIEW

This report has been reviewed and publication is approved.

Larry R. Walter
Larry R. Walter
1st Lt, USAF
DCS/Research

D. R. Eastman, Jr.
Donald R. Eastman, Jr.
DCS/Research

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ILLUSTRATIONS

NOMENCLATURE

A	Area, ft^2
d^*	Nozzle throat diameter
h	Local heat-transfer coefficient, Eq. (1)
k	Thermal conductivity
Nu	Nussult number, hx/k
Pr	Prandtl number
p	Pressure
$\dot{q}$	Heat-transfer rate, Btu/hr
R	Radius of sphere
Re	Reynolds number, $\rho ux/\mu$
r	Radius of nozzle
T	Absolute temperature
u	Velocity
x	Axial distance
β	Velocity gradient parameter, Eq. (10)
$\overline{\mu}$	Mean absorption coefficient of radiating gas
ρ	Density
σ	Stefan-Boltzmann constant

SUBSCRIPTS

aw	Adiabatic wall condition
o	Stagnation condition
ref	Reference condition
w	Wall condition
∞	Free-stream condition

1.0 INTRODUCTION

A small, continuous-flow, low-density, hypervelocity wind tunnel is in operation at the von Kármán Gas Dynamics Facility (VKF), Arnold Engineering Development Center (AEDC), Air Force Systems Command (AFSC). A description of this wind tunnel, now designated Gas Dynamic Wind Tunnel, Hypersonic (L), though formerly known as the LDH tunnel, and the results of a preliminary calibration program have been presented in Ref. 1. A recent summary of procedures for tunnel flow diagnosis is presented in Ref. 2.

Briefly, the tunnel consists of: (1) a direct-current electric arc heater, (2) a stilling chamber, (3) an axisymmetric aerodynamic nozzle, (4) a test section with model and probe traversing mechanism, and (5) a vacuum pumping system. Since the tunnel is normally operated at a stagnation temperature in the range of 3600 to 7200°R, backside water cooling is required to prevent burn-out of critical tunnel components.

Measurements of total heat-transfer rates to tunnel components under various operating conditions have been made (Ref. 3), and the results have been of use in the evaluation of the tunnel design, while also providing for a better understanding of the actual flow processes taking place in the tunnel components upstream of the aerodynamic throat. However, for the aerodynamic nozzle, where local flow properties and geometry change significantly with axial position, a measurement of the total heat-transfer rate is of only limited value. In this case, the local heat-transfer rate to the nozzle wall at a particular location and the axial variation of this quantity are of much greater interest.

Several methods for predicting the local value of the convective heat-transfer coefficient in convergent-divergent nozzles are available in the literature. Perhaps the better known of these methods, for the case of the laminar boundary layer, is that of Cohen and Reshotko (Ref. 4). This solution employs a form of the Reynolds analogy between skin friction and heat transfer. For the turbulent boundary layer, the analyses of Sibulkin (Ref. 5) and Bartz (Refs. 6 and 7) are perhaps the most widely accepted. Unfortunately, there has been a lack of appropriate experimental data to check the validity of these several methods. In a recent revision and extension of his earlier work, Bartz and his co-workers (Ref. 8) have presented some experimental data which are compared with their digital computer solutions for turbulent flow.

Manuscript received March 1964.

In the case of laminar nozzle flow, there are few published experimental data available to check the validity of the analytical predictions. The purpose of this report is to describe the experimental procedures used by the author and his co-workers to obtain measurements of the local heat-transfer coefficient in a nozzle with laminar boundary-layer flow. These experimental measurements are compared with several of the methods which can be used to predict the local heat-transfer coefficient. No new theory is presented, but an existing analytical procedure is modified to give a calculation procedure which will fit the experimental data. The results should be indicative of the heat-transfer coefficient distribution for nozzles of similar design when operating under conditions of cold wall, thick laminar boundary-layer, low-density flow.

2.0 NOZZLE DESIGN

The geometry of the water-cooled components of the wind tunnel, including the plasma torch electrodes, cylindrical settling chamber, and nozzle section, is shown in Fig. 1. For the present test, the usual nozzle was removed and replaced by the convergent-divergent nozzle described in this section.

A cross-sectional view of the aerodynamic nozzle used in this investigation is presented in Fig. 2. The internal geometry of this nozzle was selected to be identical with that of the first nozzle used in Tunnel L. The throat radius of curvature is 1 in., with a conical expansion section of 15-deg half-angle.

The external geometry of this nozzle was specially designed to facilitate the measurement of the local heat flux to finite areas along the nozzle axis. This was accomplished by machining fins of equal width to permit only radial heat conduction over the greater portion of the nozzle diameter. By this method, axial heat conduction was limited to the small areas between the nozzle internal wall and the fin roots. Individual cooling water tubes were soldered to the tip of each fin, and the heat-transfer rate through each fin was determined by measuring the water flow rate and temperature rise through each coolant tube. Convective heat transfer from the fins to the atmosphere was minimized by enclosing the complete assembly in a shroud; therefore, this mode was safely neglected.

As an alternate method for measuring the heat-transfer rate through each fin, the nozzle was designed with a thin-walled constantan sleeve as an integral part of each fin, as shown in Fig. 3. External to the

constantan sleeve was a copper sleeve; both sleeves were shrunk fit onto the main copper body before final machining of the fins. The two interfaces between the constantan sleeve and the adjacent copper were expected to act as thermocouple junctions, enabling an "average" temperature to be measured at each interface. With these two temperatures known, the heat flux through a fin could be computed; the thermal conductivity and wall thickness of the constantan sleeve are known quantities. Small holes were drilled into each fin to allow special thermocouple leads to contact the proper constantan or copper material.

The complete device was designed to fit on the total mass flow calorimeter which has been used for other heat-transfer and flow calibration experiments. A complete description of this calorimeter is given in Ref. 3. Photographs of the present installation are presented in Figs. 4 and 5.

3.0 EXPERIMENTAL RESULTS

3.1 FLOW CONDITIONS

The experimental data presented herein were obtained over a range of flow conditions most commonly used in Tunnel L. Mass flow rate and total temperature or enthalpy, with associated variations in total pressure, were judged to be the most significant independent variables in this study. Therefore, data were obtained at three total temperatures for each of three mass flow rates for a total of nine flow conditions. Nitrogen was used as the working fluid.

It has been shown (Ref. 2) that the gas, which is highly energetic as it leaves the plasma torch, will approach equilibrium conditions before entering the nozzle if the settling section upstream of the nozzle is of sufficient length. A nominal section length of 5 in., which is sufficient to assure recombination and chemical-kinetic equilibrium for these flow conditions, was used.

3.2 METHOD OF CALCULATION

The raw data for each run consisted of the measured flow rate and temperature rise of the water in each fin coolant tube and the measured temperatures at the copper-constantan interfaces of each fin. The flow rates were determined with the use of a known volume and a stopwatch. The water temperatures were measured by copper-constantan thermocouple junctions inside the tubes. From these data the rate of heat flow through each fin was computed by the two independent methods.

The heat-transfer coefficient is conventionally defined by the equation

$$h = \frac{\dot{q}}{A_w (T_{aw} - T_w)} \tag{1}$$

for high speed flow. The driving potential is taken as the difference between the adiabatic wall temperature and the actual wall temperature. The adiabatic recovery factor was taken as $(Pr)^{1/2}$, so that

$$\frac{T_{aw} - T_\infty}{T_o - T_\infty} = r = (Pr)^{1/2} \tag{2}$$

Reinecke (Ref. 13) shows that $(Pr)^{1/2}$ is an accurate expression for the recovery factor in laminar flow even in favorable or adverse pressure gradients.

In all calculations the wall temperature was taken as 600°R. The actual wall temperature was slightly above this value upstream of the throat and slightly below this value downstream of the throat. Use of this constant wall temperature introduced an insignificant error in the calculations since the adiabatic wall temperature was always comparatively large, being between 4500 and 6500°R.

In all calculations the flow was assumed to be vibrationally frozen over the entire length of the nozzle. This assumption is in substantial agreement with the experimental results presented in Ref. 2 as well as a recent theoretical analysis (Ref. 14) which is to be published.

3.3 RESULTS

The experimental results are presented in the form of a plot of observed heat-transfer coefficient versus axial location along the nozzle for each flow condition. These results are presented in Figs. 6a through i. The open symbols represent data obtained from the water-flow measurements, while the solid symbols represent the results of the special thermocouple measurements.

Although the results obtained by the two independent methods indicate the same trend, they differ enough that a critical evaluation should be made to determine which method is more accurate. The special thermocouples generally give lower results upstream of the throat and higher results downstream of the throat. This method is based on the unproven technique of allowing a large surface interface area to act as a thermocouple junction. In the event of a non-uniform temperature at the interface, the indicated temperature may be a type of "average" temperature. The problem might be further aggravated by a non-uniform surface

contact along the interface. Also, when the heat-transfer rate is small, the calculations involve the difference between two relatively large numbers subject to the above inaccuracies. Furthermore, since the water-flow method gives higher results upstream of the throat where the heat-transfer rate is large, and since any heat losses would tend to reduce the measured values, it follows that the correct values must be at least as great as those given by the water-flow measurements. On the basis of these arguments, it is concluded that the water-flow measurement system produced the more reliable results. These results will be used later for comparison with theory.

The measured heat-transfer coefficient appears to reach a maximum value slightly upstream of the throat and falls off sharply immediately downstream of the throat. Despite the fact that the nozzle was designed to minimize axial heat conduction through the walls, some heat conduction axially below the fin roots is inevitable. This has the effect of "smoothing out" the measured heat-transfer coefficient, especially in the throat region. Rough calculations which take into account this axial heat conduction indicate that the "true" heat-transfer coefficient at the first station upstream of the throat may be as much as 30 percent higher than measured and that the coefficients at adjacent stations should be adjusted by an appropriate amount. Therefore, the "true" heat-transfer coefficient exhibits a sharper peak than the measured values would indicate. However, for practical design purposes where backside cooling requirements are of main importance, the measured values should be of more interest and utility. In the following section the emphasis will be on predicting the "measured" heat-transfer coefficient which is applicable for backside cooling purposes.

4.0 THEORETICAL ANALYSES

One of the main objectives of this investigation was to examine several analytic procedures which may be used to predict local heat-transfer rates in laminar flow nozzles with highly cooled walls. Those methods which will be considered here are the simple flat plate equation, the incremental flat plate technique of Pasqua and Stevens (Ref. 9), the method of Cohen and Reshotko, and a simplification of the method of Beckwith and Cohen (Ref. 10).

4.1 FLAT PLATE METHOD

In the absence of more appropriate information, it has often been expedient to use the well-known flat plate equation

$$Nu_w = 0.332\ Re_w\ (Pr)^{1/3} \tag{3}$$

to approximate the heat transfer in a nozzle. This equation, rederived in Ref. 9, is strictly applicable for uniform flow of constant properties external to the boundary layer over a flat plate of constant wall temperature. Although the compressible flow of a highly heated gas through a nozzle fails to satisfy these conditions, this equation should be useful as a first approximation for nozzle flow. The local heat-transfer coefficient, calculated from this equation assuming a Prandtl number of 0.72, is presented in Fig. 7 for a typical flow condition. Far downstream of the nozzle throat, the agreement with the experimental data is good. In the throat region, however, where free-stream properties are rapidly changing, the equation underestimates the heat-transfer coefficient. The successful downstream results suggest the possibility of a modified flat plate equation which could be used to predict heat-transfer rates near the throat.

The flat plate methods depend on the choice of a starting point for the axial distance found in the Reynolds number. In all calculations the beginning of the convergent section was taken as the starting point. Although the results could be altered slightly by choosing a different starting point, the present choice offers the best defense on purely geometric considerations.

All fluid properties used in the calculations were evaluated at the nozzle wall temperature. An alternate procedure is the well-known reference temperature method of Eckert (Ref. 12). As a matter of interest, the heat-transfer coefficient at the throat for a typical flow condition was calculated using the simple flat plate equation and the reference temperature method. This resulted in a 10-percent reduction in the predicted value, compared with the results using fluid properties at the wall temperature. Since this effect was in the opposite direction of the desired change, the reference temperature method was not used.

4.2 INCREMENTAL FLAT PLATE METHOD

Pasqua and Stevens (Ref. 9) have proposed an incremental flat plate technique to account for changes in free-stream properties which may affect the heat-transfer coefficient. They represent the nozzle by a series of incremental flat plates of length Δx. With each increment there is associated a change in the flat plate heat-transfer coefficient, Δh. The variation in heat-transfer coefficient across an incremental flat plate Δx_n is represented as

$$h_{n+1} = h_n + \Delta h_n \tag{4}$$

The increment Δh_n is assumed to be a function of x, ρ, and u so that

$$\Delta h_n = \left.\frac{\partial h}{\partial x}\right)_{u,\rho} \Delta x_n + \left.\frac{\partial h}{\partial u}\right)_{x,\rho} \Delta u_n + \left.\frac{\partial h}{\partial \rho}\right)_{x,u} \Delta \rho_n \tag{5}$$

The partial derivatives are obtained from Eq. (3).

$$\left.\frac{\partial h}{\partial x}\right)_{u,\rho} = - \left[\frac{0.332}{2} (Pr)^{1/3} k\right]_n (Re)_n^{1/2} x_n^{-2} \tag{6}$$

$$\left.\frac{\partial h}{\partial u}\right)_{x,\rho} = \left[\frac{0.332}{2} (Pr)^{1/3} k\right]_n (Re)_n^{1/2} (u_n x_n)^{-1} \tag{7}$$

$$\left.\frac{\partial h}{\partial \rho}\right)_{x,u} = \left[\frac{0.332}{2} (Pr)^{1/3} k\right]_n (Re)_n^{1/2} (\rho_n x_n)^{-1} \tag{8}$$

This method requires that the heat-transfer coefficient at some starting point be known. In their report in the absence of any experimental data, Pasqua and Stevens (Ref. 9) chose a stagnation point at the nozzle entrance as a starting point, following the lead of Cohen and Reshotko (Ref. 4). In the actual nozzle the flow near the nozzle exit best satisfies the conditions required for flat plate flow. Here the axial changes in velocity and density are small compared with changes in other parts of the nozzle. Furthermore, the present experimental data and the ordinary flat plate equation agree closely at this part of the nozzle. Therefore, it appears that the best starting point for the incremental flat plate method is at the nozzle exit, with the heat-transfer coefficient at this point calculated using the ordinary flat plate equation.

The incremental flat plate method results are also shown in Fig. 7 for comparison with the ordinary flat plate calculation and the experimental data. The method compares favorably with the experimental data between the nozzle throat and the exit. Upstream of the throat the experimental data are still slightly higher than the curve representing the incremental technique. This fact will be the subject of further discussion.

4.3 COHEN AND RESHOTKO METHOD

The method of Cohen and Reshotko (Ref. 4) is possibly the best known procedure for the calculation of the compressible laminar boundary layer with heat transfer and arbitrary pressure gradient. Colleagues of the present author have incorporated a correction for transverse curvature and have programmed this method for the digital computer.

These computer solutions were used to calculate the local heat-transfer coefficient by the method of Ref. 4. The results are presented for a typical flow condition in Fig. 8. The Cohen and Reshotko method predicts a heat-transfer coefficient much higher than the experimental values, especially in the throat region.

It has been pointed out that, because of axial conduction, the "true" heat-transfer coefficient near the throat may be somewhat greater than the measured values indicate. It is possible to make a rough calculation to account for this axial conduction in the throat region. Using the temperature measured at the copper-constantan interface of each fin and the measured heat conducted through each fin, the temperatures at the fin roots may be calculated. Assuming that the temperature difference between adjacent fin roots is the same value as the driving potential for axial heat transfer in the vicinity of the fin roots, the axial heat transfer from station to station can be calculated. An energy balance at each station then gives the heat transferred to the nozzle walls at each station.

The results of these approximate calculations are also presented in Fig. 8 for comparison with the Cohen and Reshotko prediction and the measured results. Although the corrected values exhibit a sharp peak near the throat, the magnitude of these values is still far below the Cohen and Reshotko prediction. Furthermore, downstream of the throat where axial conduction is negligible, the Cohen and Reshotko prediction is about double the measured values, even though the magnitude of the difference is small. It is evident that the Cohen and Reshotko prediction is too large, even when the experimental results are corrected for axial conduction.

4.4 MODIFIED SIMILAR SOLUTION

A fourth analytic procedure which was studied was a modification and simplification of an existing boundary-layer similar solution. Beckwith and Cohen (Ref. 10) have developed a method for the calculation of heat-transfer distributions on yawed cylinders of arbitrary cross-sectional shape and on bodies of revolution in high-speed flows with laminar boundary layers. Briefly, the method is designed to satisfy the integral energy equation with the assumption of local similarity, wherein the actual boundary-layer profiles at every station are replaced by corresponding profiles from a family of similar solutions. This method, which uses the energy equation, should be better suited for heat transfer than the Cohen and Reshotko method, which uses the momentum integral equation in a somewhat analogous approach.

Inasmuch as the Beckwith-Cohen analysis was not specifically designed to cover the internal flow heat-transfer problem, a number of simplifying assumptions in the calculation procedure have been made. Justification of these assumptions will rest on the final results. Using the assumption of unit Prandtl number and approximations appropriate in cold-wall cases given in their report, plus an additional assumption concerning the stagnation enthalpy profile along the nozzle, the results are reduced to an equation of the form

$$\frac{h}{h_{ref}} = \left[\frac{p}{p_{ref}} \; \frac{(T/T_o)_{ref}}{(T/T_o)} \; \frac{\beta_{ref}}{\beta} \; \frac{\left(\frac{du}{dx}\right)}{\left(\frac{du}{dx}\right)_{ref}} \right]^{1/2} \left[\frac{(T_{aw} - T_w)_{ref}}{(T_{aw} - T_w)} \right] \tag{9}$$

which gives the ratio of the heat-transfer coefficient at any location to the heat-transfer coefficient at some reference location. The velocity-gradient parameter is defined by the equation

$$\beta = \frac{2}{u^2} \; \frac{du}{dx} \; \frac{T_o}{T} \; \frac{1}{pr^2} \int_o^x pur^2 \, dx \tag{10}$$

Here, again, the method requires that the heat-transfer coefficient at some reference point be known. As before, it appears that the best reference location is the nozzle exit, with the heat-transfer coefficient at this point calculated using the simple flat plate equation.

The results of these calculations are presented in Figs. 9 through 13. Also shown are the incremental flat plate results and the experimental results. The similar solution technique gives a distribution which is much like the distribution based on the incremental technique although it peaks to a somewhat higher value at the nozzle throat.

The incremental flat plate method and the Beckwith-Cohen solution are shown to give the best predicted results for the nozzle tested. The practical decision of which method to use in a particular situation involves several considerations. The incremental flat plate method requires only that the velocity and density or pressure distributions be known for each flow condition considered. The pressure gradient and velocity gradient do not enter into the calculations. Therefore, the method is simple to use, especially when only a single flow condition is considered.

When results for more than one flow condition are needed for the same nozzle, the similar solution becomes very attractive. Even though the velocity gradient distribution is required, the few additional calculations are relatively minor because the dimensionless heat-transfer

distribution, obtained from Eq. (9), is the same for any flow condition through a particular nozzle. The actual distribution for each flow condition is then determined from the simple flat plate calculation at the nozzle exit.

4.5 DISCUSSION OF ANALYTIC RESULTS

Downstream of the throat both the incremental flat plate and the Beckwith-Cohen similar solutions display excellent agreement with the experimental data when the heat-transfer coefficient calculated by use of the simple flat plate equation at the exit is used as the reference or starting point. Upstream of the throat the two solutions predict values lower than the experimental results. Also, the experimental results upstream of the throat display significantly different trends from run-to-run. Possible reasons for this apparent anomaly upstream of the throat should now be mentioned.

The most obvious reason for the discrepancy between the experimental data and the analytic predictions upstream of the throat is the absence of a truly valid flow model in this region. All values used in the calculations upstream of the throat have been computed from one-dimensional gas dynamic equations. It is likely that the one-dimensional flow model deviates from the actual flow upstream of the throat. In that case the velocities near the converging wall could be somewhat greater than those calculated from the one-dimensional flow model. This could well account for the higher-than-predicted heat-transfer rates.

A second consideration is the effect of thermal radiation to the converging nozzle walls. The working gas leaves the plasma torch in a highly non-equilibrium state with an average temperature of the order of 12,000°R. Heat losses in the settling chamber reduce the gas temperature to the equilibrium stagnation values quoted in this report by the time the gas enters the nozzle. If the gas in the settling chamber radiates a significant quantity of thermal energy to the nozzle walls, the measured heat-transfer coefficient would be greater than the predicted results, which do not include the thermal radiation effects.

The thermal radiation from a gas to a portion of its enclosure is, in general, difficult to calculate. The calculation becomes more difficult in the entrance region of a nozzle because of the complexity of the geometry involved. Further, in the present case where the gas may not be in a state of equilibrium and where large temperature gradients are known to exist, it is extremely difficult to determine which values of fluid properties should be used in the calculations.

To calculate the thermal radiation to the walls of a nozzle entrance, the assumption of a spherical volume of emitting gas was considered in Refs. 9 and 11. The solution of Pasqua (Ref. 9) reduces to the equation

$$\frac{\dot{q}}{A} = \frac{4}{3}\,\bar{\mu} R \sigma T^4 \qquad (11)$$

which gives the heat transfer per unit area from an emitting and non-absorbing gas occupying a sphere of radius R. The mean absorption coefficient, $\bar{\mu}$, is found by integrating the intensity per unit wavelength over all wavelengths.

To demonstrate the potential effect of thermal radiation on the measured heat-transfer coefficient upstream of the throat, the following calculation is presented:

Consider that the radiating volume is a sphere of radius 0.25 in., approximately the size which would fit into the nozzle tangent to the surface at the location corresponding to the first measurement. A reasonable value for the mean absorption coefficient, based on the results of Ref. 9 for air, is 0.01 cm^{-1}. Using Eq. (11), the following increases in heat-transfer coefficient are calculated for radiation from the sphere at several temperatures:

Temperature, °R	Δh, $\frac{Btu}{hr\text{-}ft^2\text{-}^\circ F}$
6000	3.5
7000	6.4
8000	11.0
9000	17.7
10000	26.9
11000	39.3
12000	55.6

The effect of the fourth power of temperature is quite evident in these calculations. The values of Δh would also increase linearly as the size of the radiating sphere increased.

An additional variable in the thermal radiation process is the surface absorptivity of the nozzle. In Eq. (11) the nozzle is assumed to be a perfect absorber, while emitting essentially no radiation because of its low temperature. If the wall is not a perfect absorber, the energy it absorbs will be reduced from that calculated from Eq. (11). However, it is doubtful whether a partially reflective surface could be maintained because of surface contamination. In fact, the probable effects of

surface contamination are visible in the experimental data. The data runs are numbered consecutively in the order in which they were observed. In the first run, the experimental data upstream of the throat are in fair agreement with the incremental flat plate and similar solution predictions. For the following runs, the trend is toward a continually increasing discrepancy between the observed and predicted results. This trend is thought to be the effect of surface contamination resulting in an increase in absorptivity at the nozzle surface.

The effect of another variable on the heat-transfer rates to the converging portion of the nozzle has been previously documented. Figure 14, which is taken from Ref. 3, presents the total heat loss to the convergent section for a number of flow conditions with plasma jet anode diameter as the independent variable. The rather surprising increase in heat loss as the anode diameter is decreased has not been fully explained. Since no particular effort was made to ensure closely standardized anode replacements during the present series of experiments, the data may reflect a run-to-run variation in the results upstream of the throat caused by routine anode replacement. Since all anodes normally used are of similar configuration, this effect is probably of small importance in the present data as compared to the effects of thermal radiation.

The above considerations show the futility of obtaining a suitable correction term which could be used to predict the heat-transfer coefficient upstream of the throat under all circumstances. A reasonable lower limit in this region appears to be the prediction by the incremental flat plate technique or the similar solution. A reasonable upper limit, based on the experimental data together with the radiation calculation using the gas temperature at the plasma torch, appears to be the maximum value obtained from the similar solution. The actual, effective heat-transfer coefficients should fall between these two limits, with the results heavily dependent upon the factors involved in thermal radiation.

5.0 CONCLUSIONS

5.1 EXPERIMENTAL RESULTS

The experimental results presented in this report should be indicative of the heat-transfer coefficient distribution for cold-wall nozzles of similar design when operating with highly heated low-density flow. The high heating rates in the immediate vicinity of the nozzle throat are evident.

The design of the experimental nozzle proved to be satisfactory for obtaining the heat-transfer coefficient distribution. Although the individual calorimeters on each fin provided the better data, the special thermocouple junctions within each fin provided a useful method of obtaining supporting data.

5.2 THEORETICAL ANALYSES

It was demonstrated that the simple flat plate equation, which has often been used in the absence of better information, seriously underestimates the heat-transfer coefficient in all parts of the nozzle except near the exit. At the exit, however, where the Mach number was about 5.7 in these tests, the flat plate equation gave excellent results.

The incremental flat plate method accounts for some of the shortcomings of the simple flat plate equation. When the nozzle exit value obtained from the simple flat plate equation is used as the starting condition, the incremental flat plate method provides good results downstream of the throat.

The failure of the Cohen and Reshotko method is somewhat surprising in view of the success of the method in predicting skin friction and boundary-layer properties in similar nozzles. A plausible explanation for these results is that the method is designed to satisfy the momentum integral equation rather than the energy integral equation.

The modified similar solution of Beckwith and Cohen provides good agreement with the experimental data at the throat and downstream if, as shown earlier, it is referenced to the flat plate calculation at the nozzle exit. Because of the sharp peak at the throat, this method should be a better representation of the "true" heat-transfer coefficient at the nozzle surface, before the "smoothing out" effects of axial conduction have occurred.

Upstream of the throat, the difficulties involved in predicting the measured results are attributed to possible inaccuracy of the assumed one-dimensional flow model and the effects of thermal radiation. Nozzle wall contamination is suggested as a possibly important factor affecting heat-transfer rates in the convergent section, acting to decrease the surface reflectivity and increase the energy absorbed for a given radiation intensity.

After the above statements are considered and in view of the slightly more conservative results near the nozzle throat, it is recommended

that the Beckwith-Cohen procedure with the modifications discussed be used to predict the local heat-transfer coefficient in the throat region of a nozzle with high speed laminar flow.

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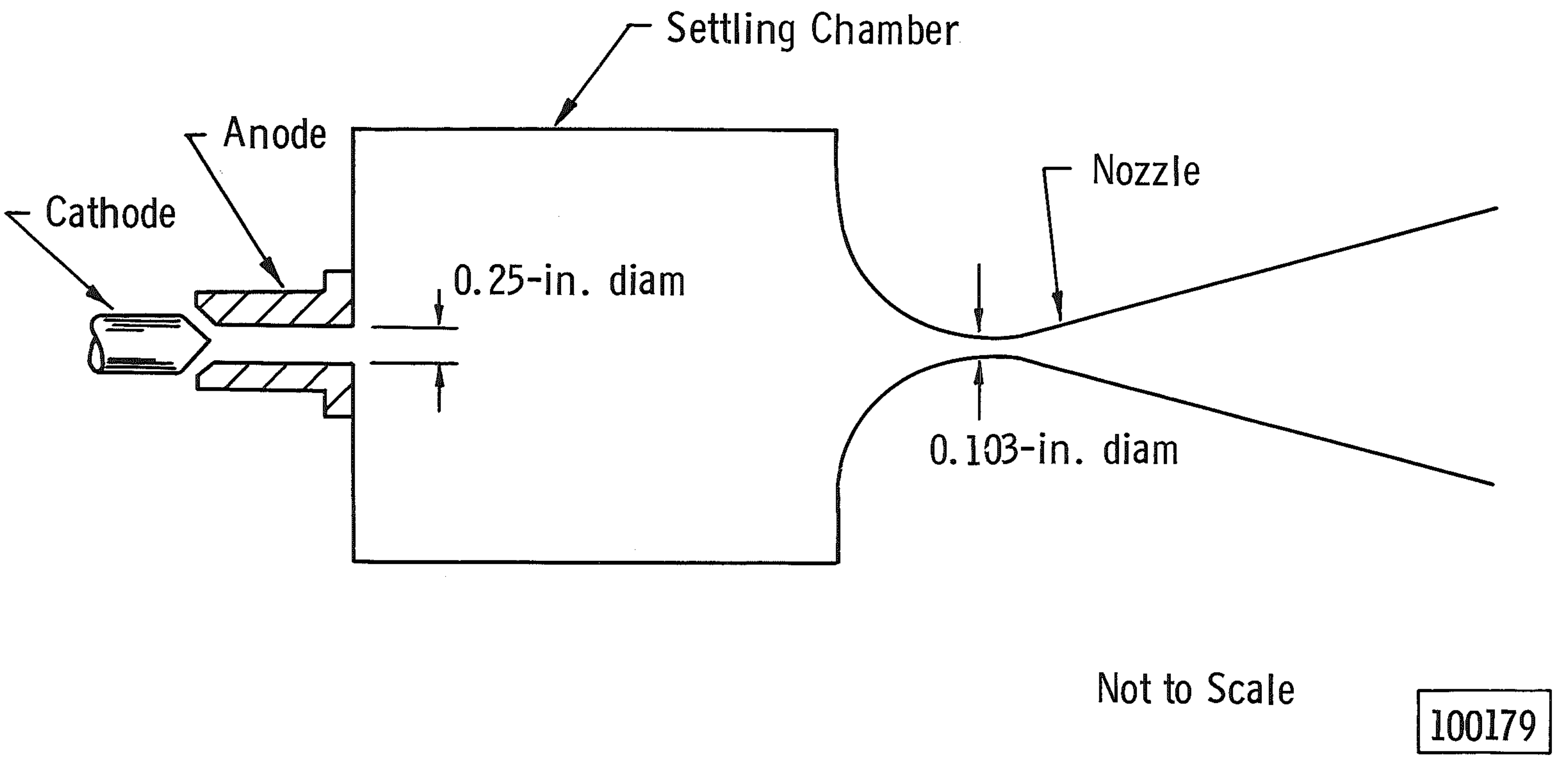


Fig. 1 Schematic of Typical Electrode-Settling Chamber-Nozzle Configuration

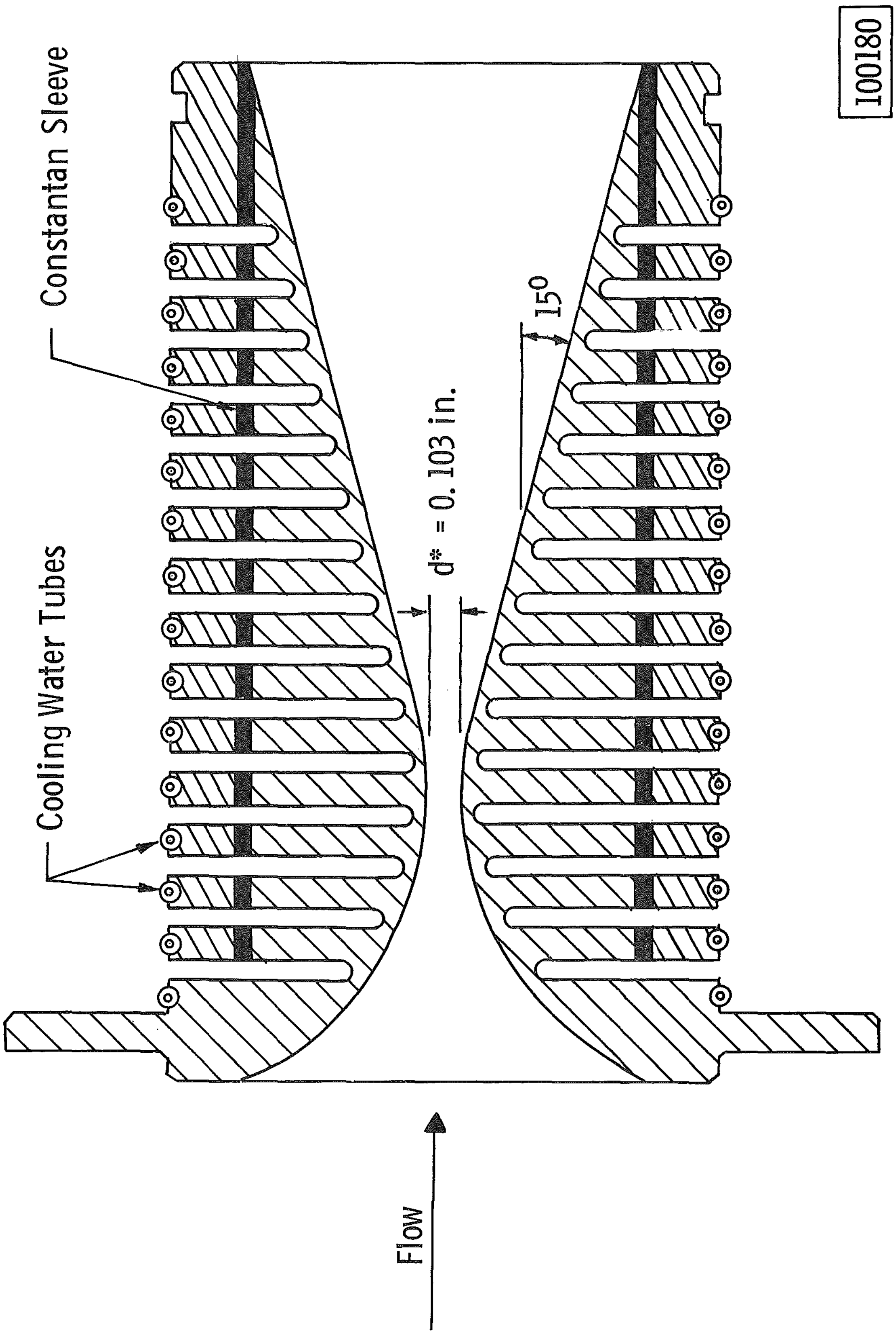


Fig. 2 Sectional View of Experimental Nozzle

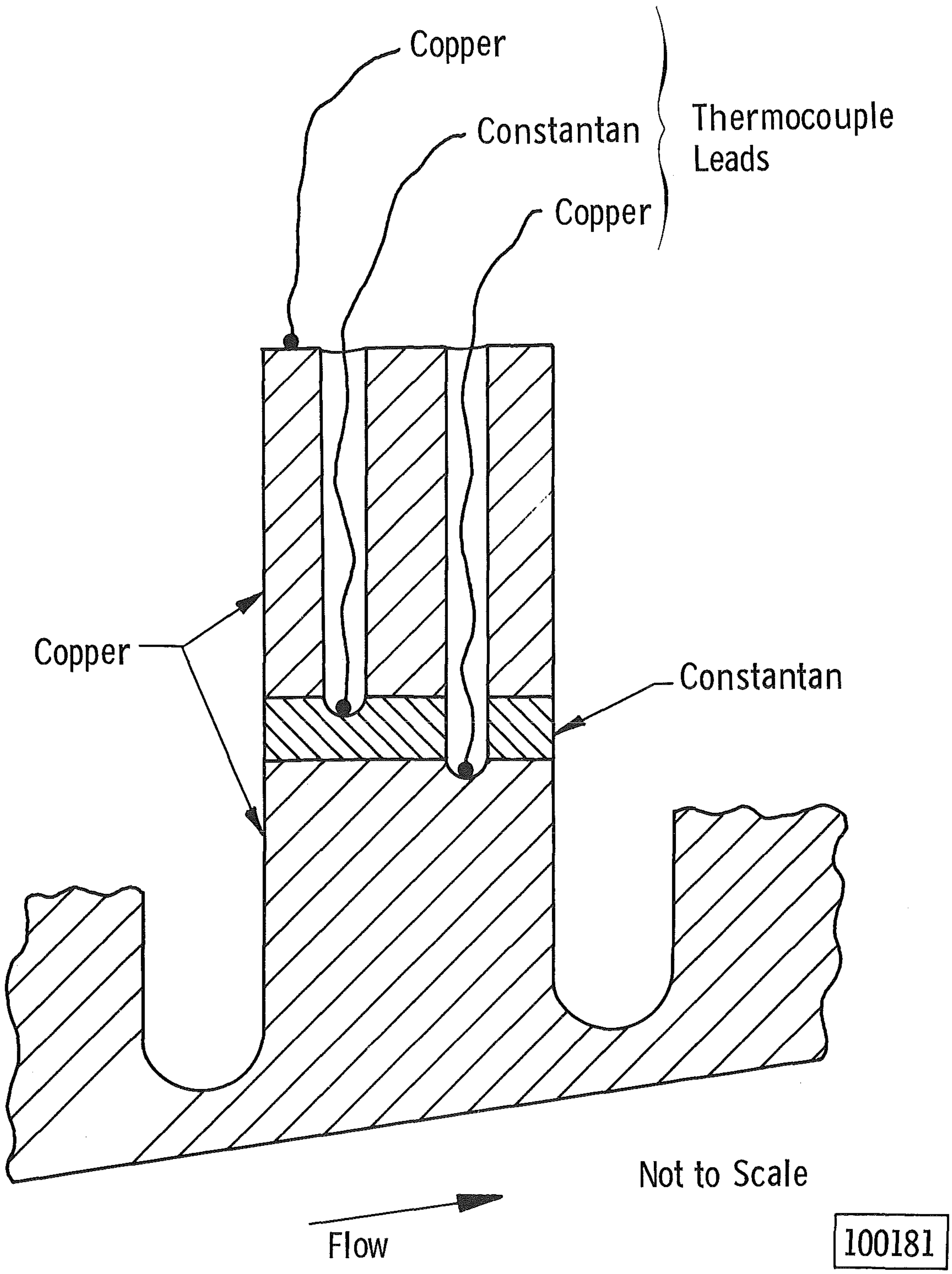


Fig. 3 Detail of Constantan Sleeve and Special Thermocouples in Each Fin

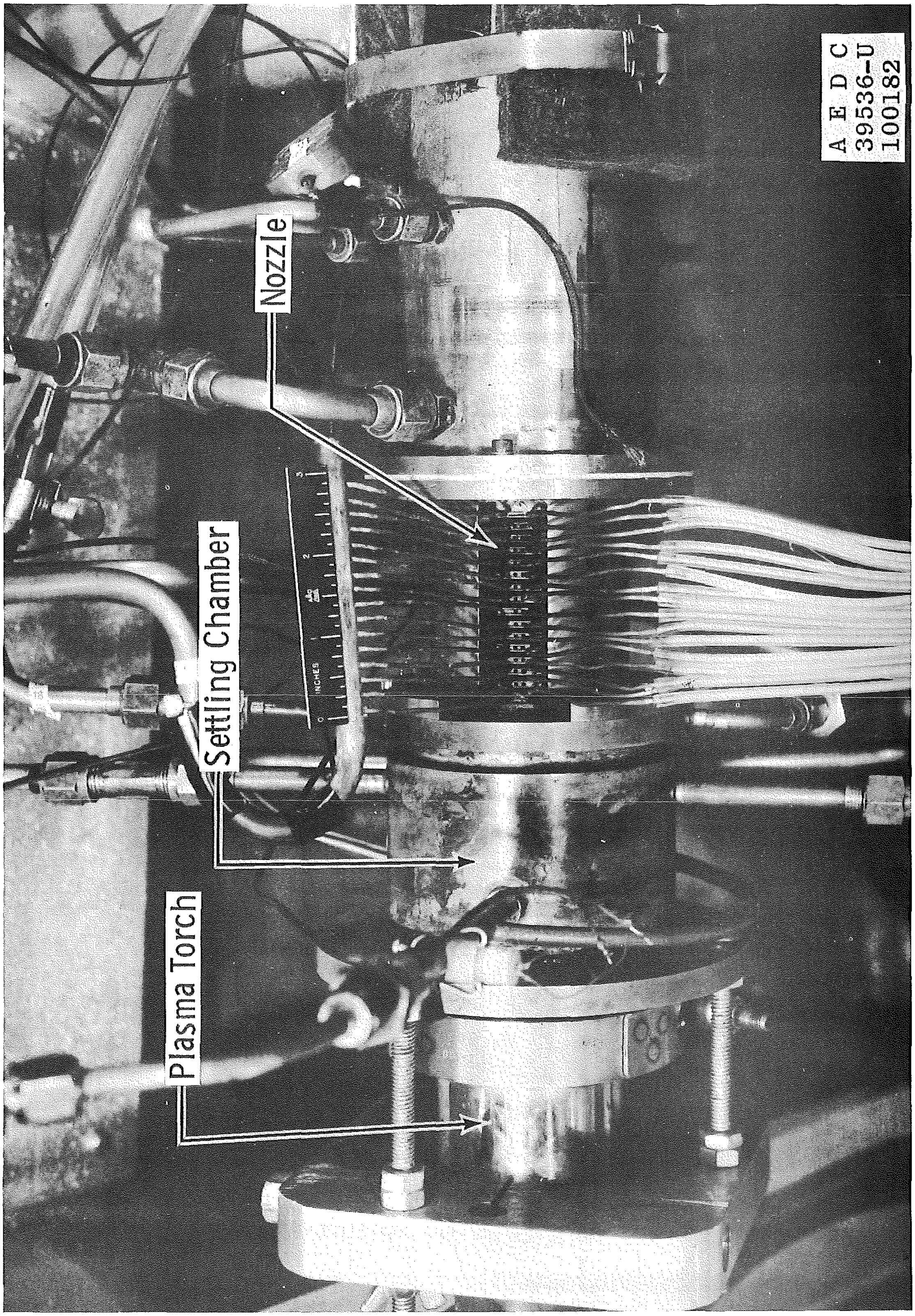


Fig. 4 Nozzle Installation

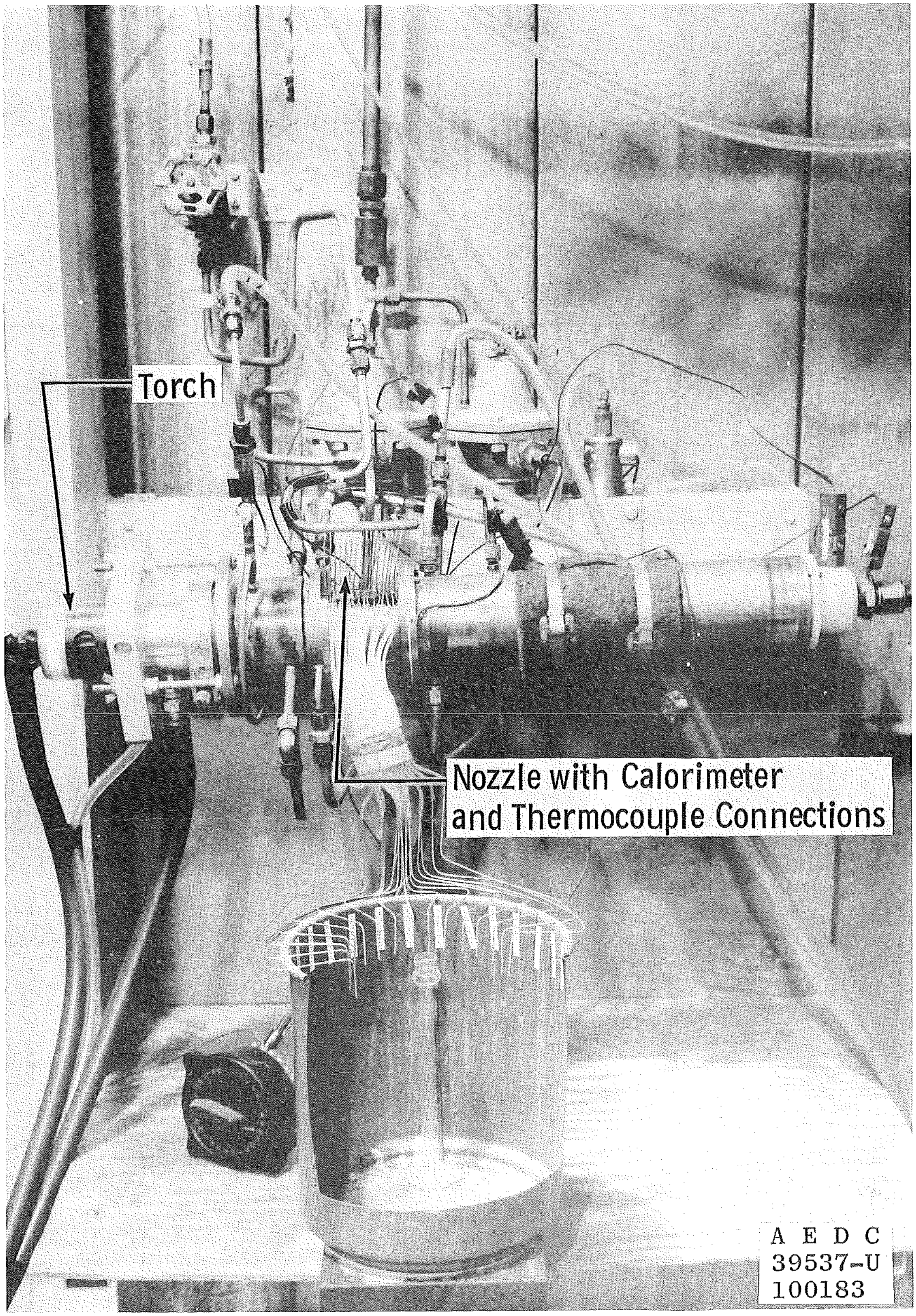
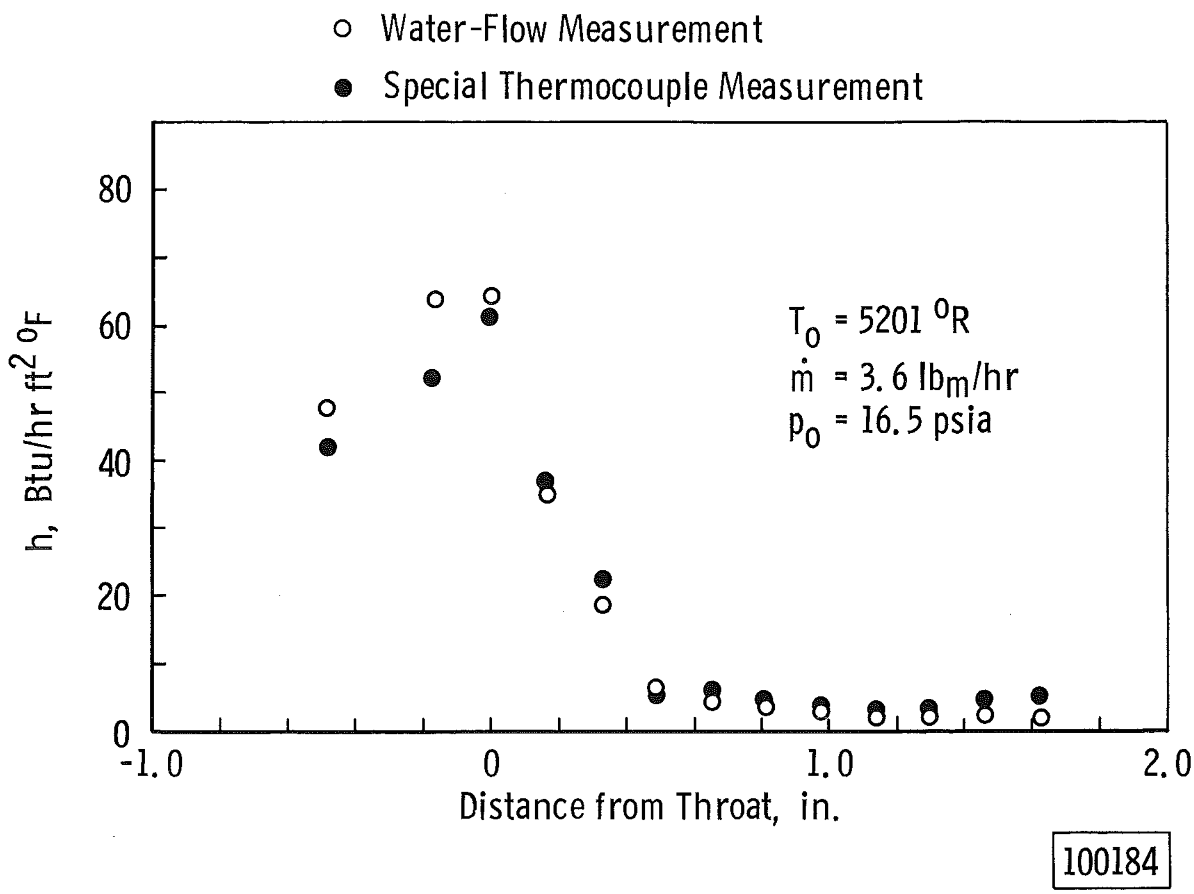
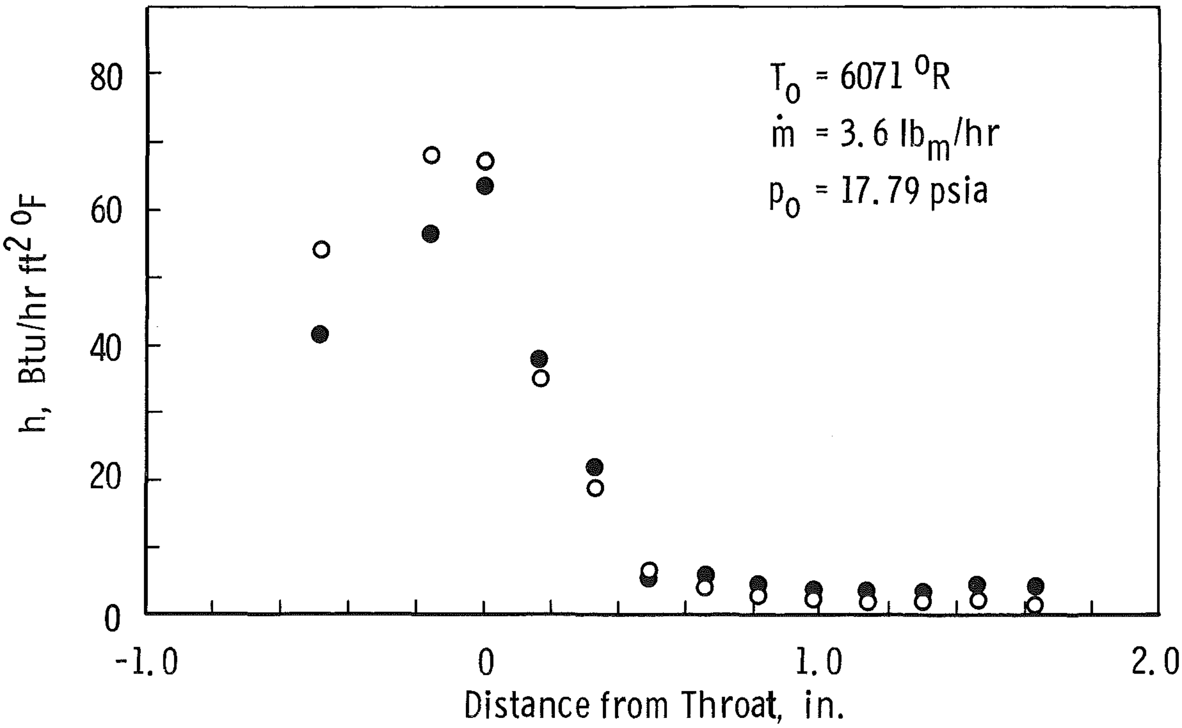


Fig. 5 Calorimeter Installation

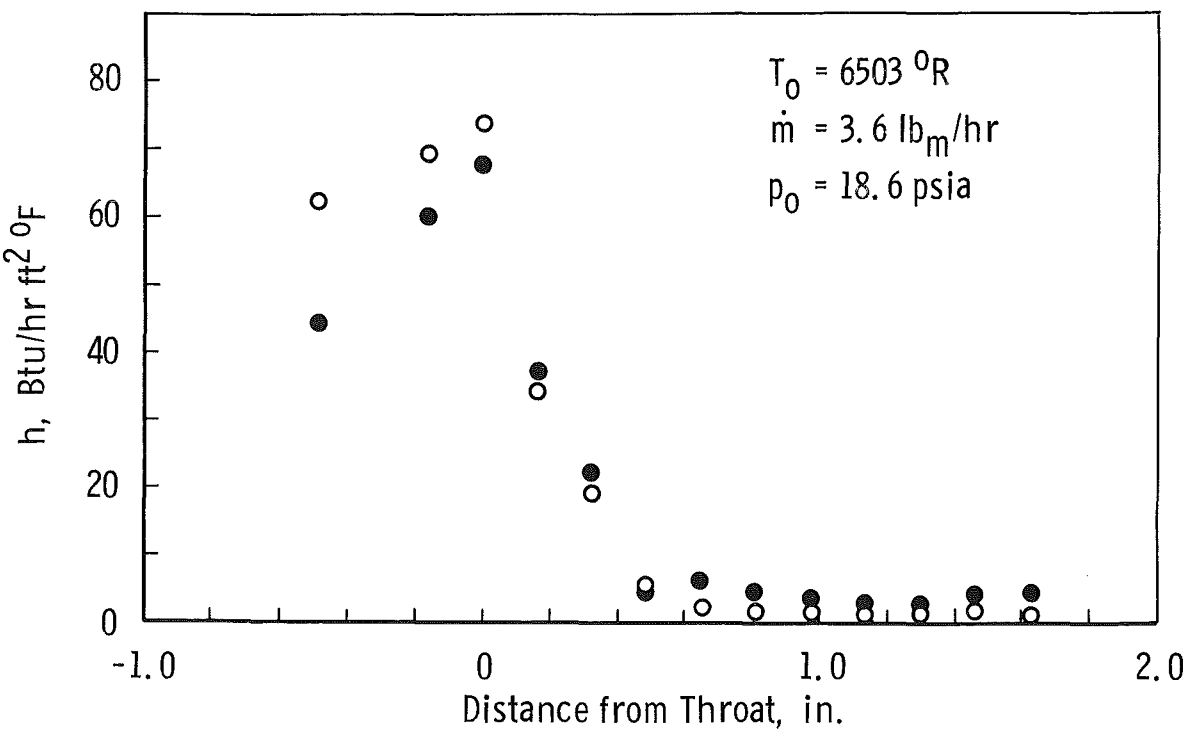


a. Run No. 1

Fig. 6 Experimental Results with Nitrogen



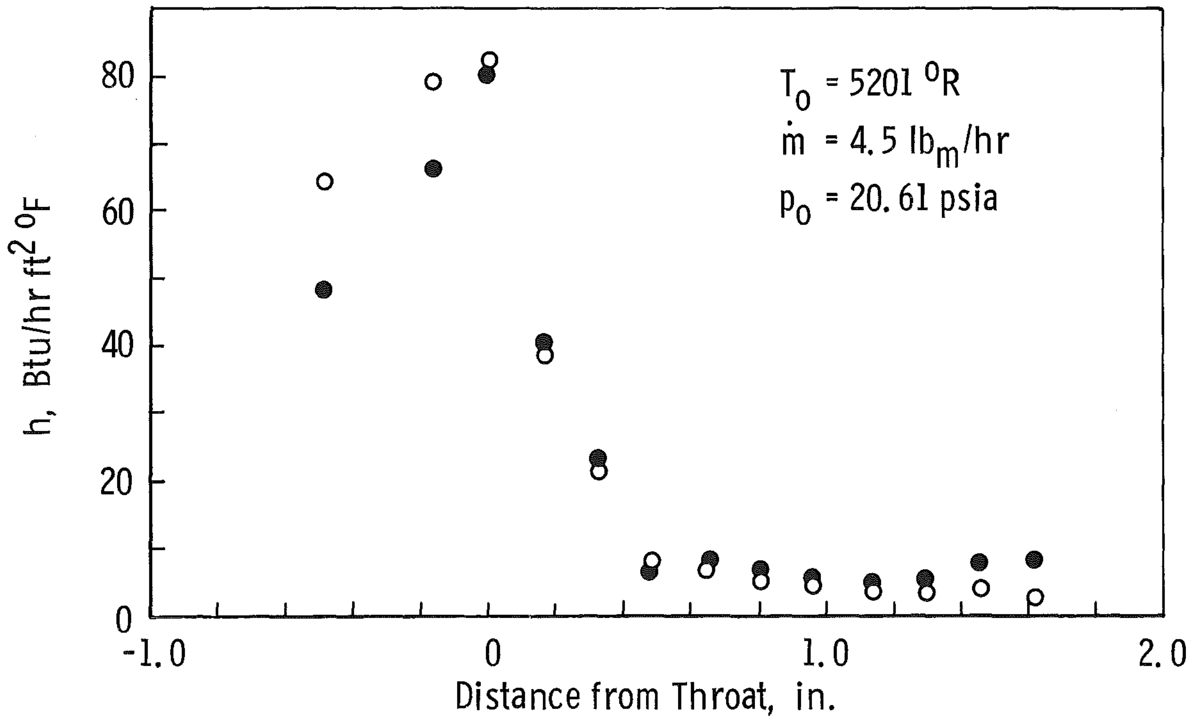
b. Run No. 2



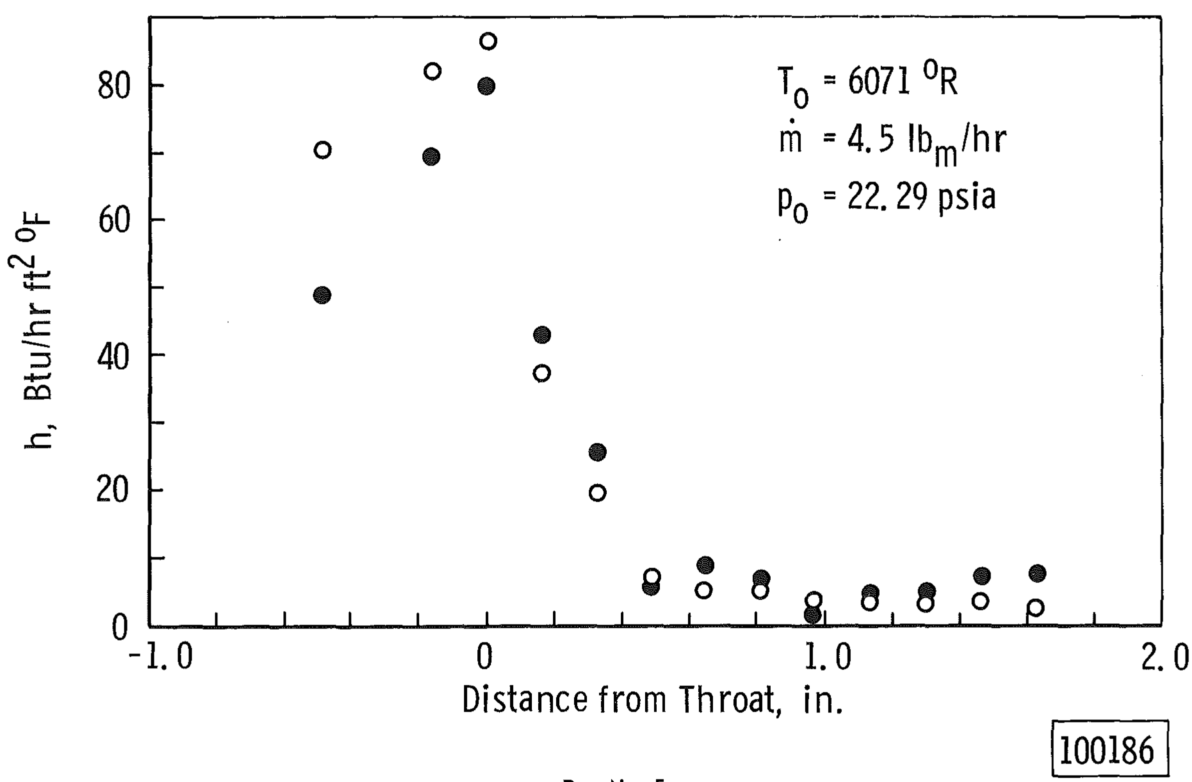
c. Run No. 3

100185

Fig. 6 Concluded

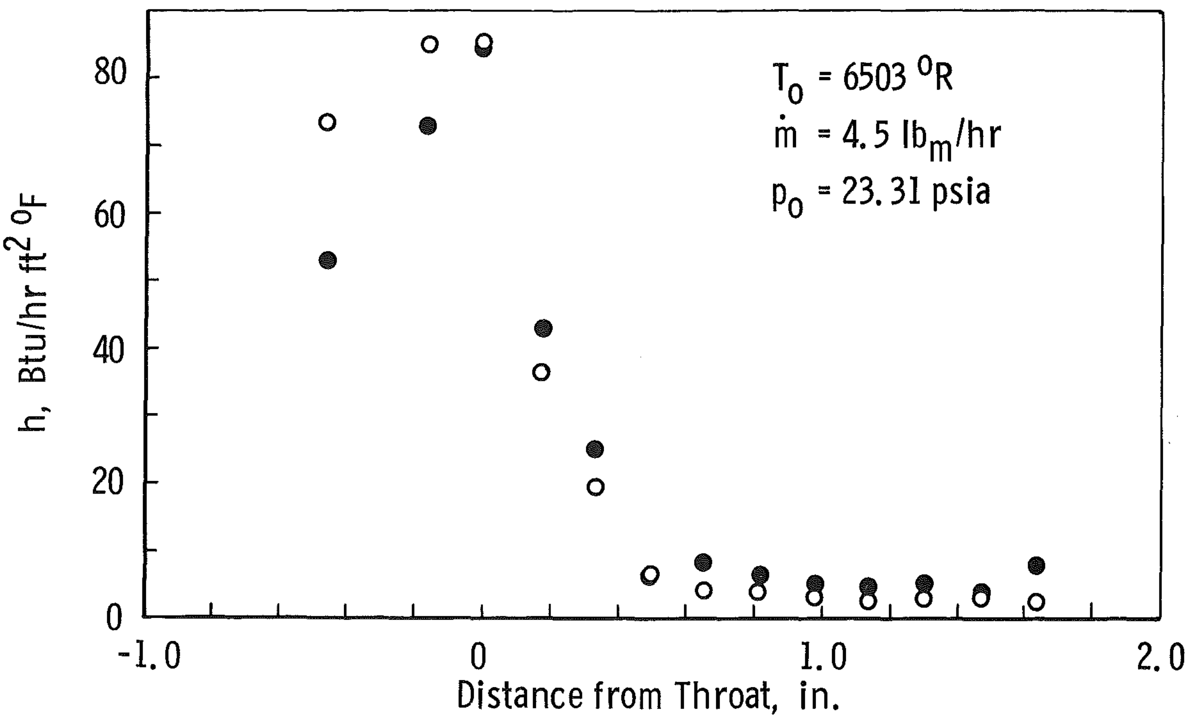


d. Run No. 4

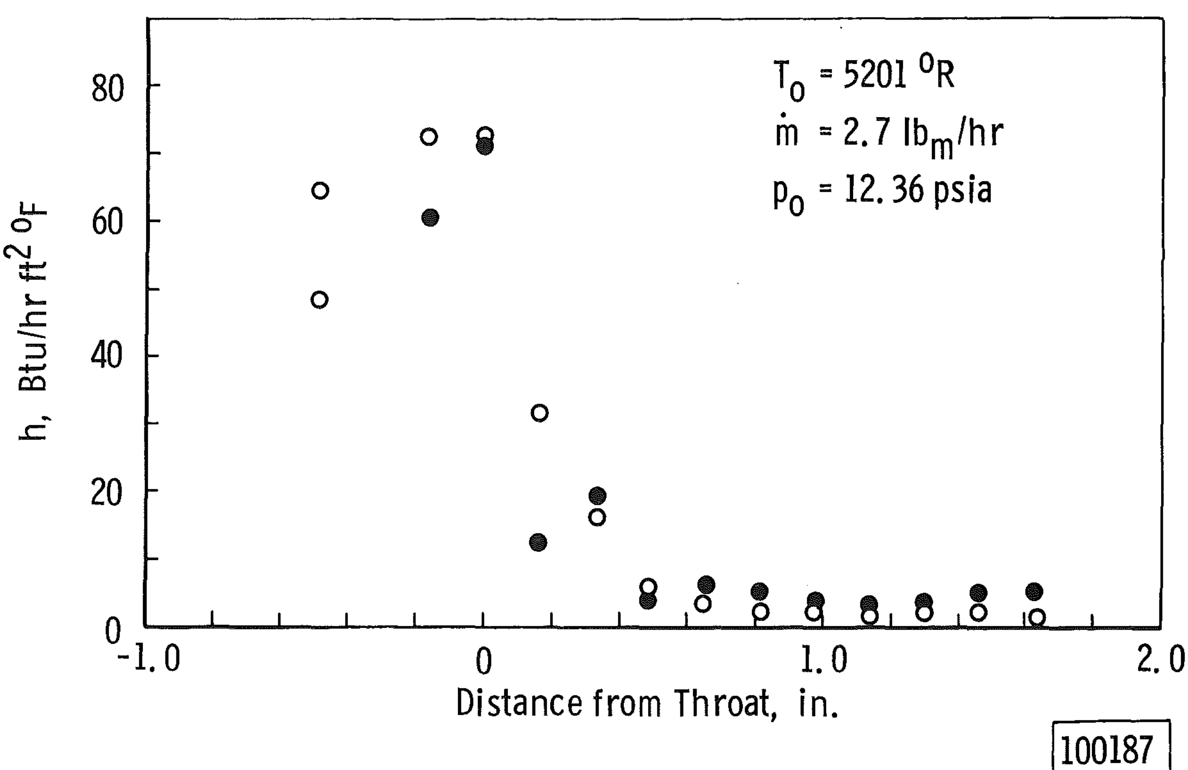


e. Run No. 5

Fig. 6 Continued

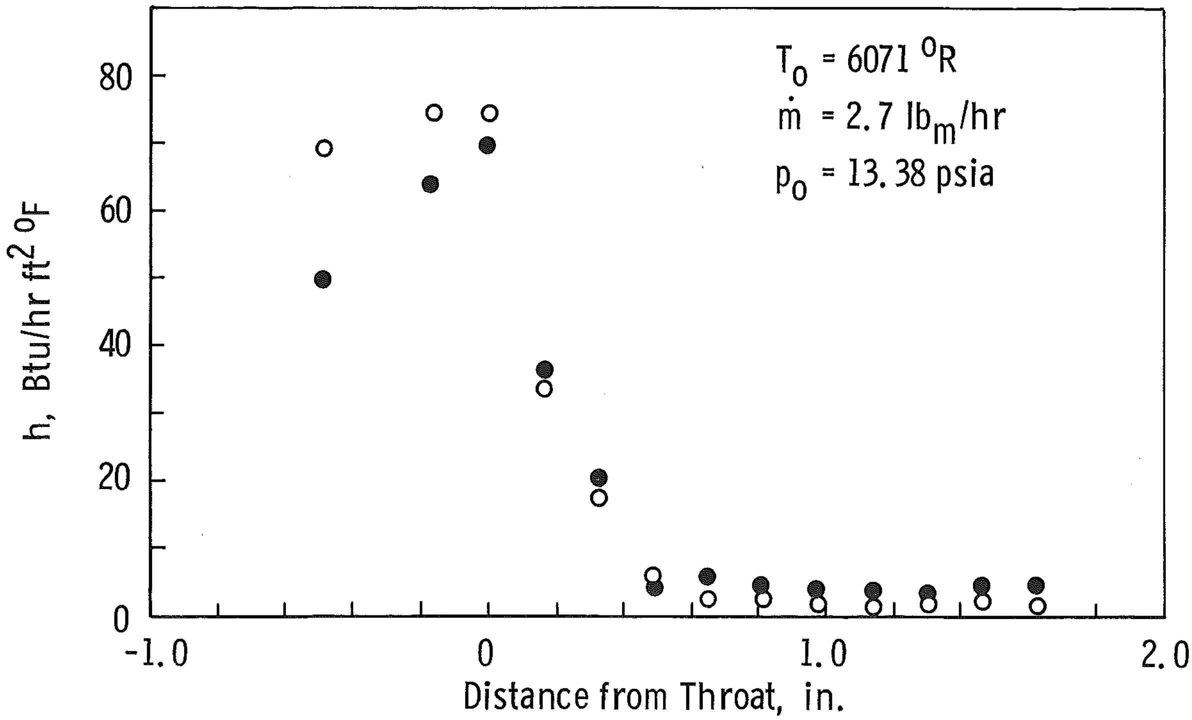


f. Run No. 6

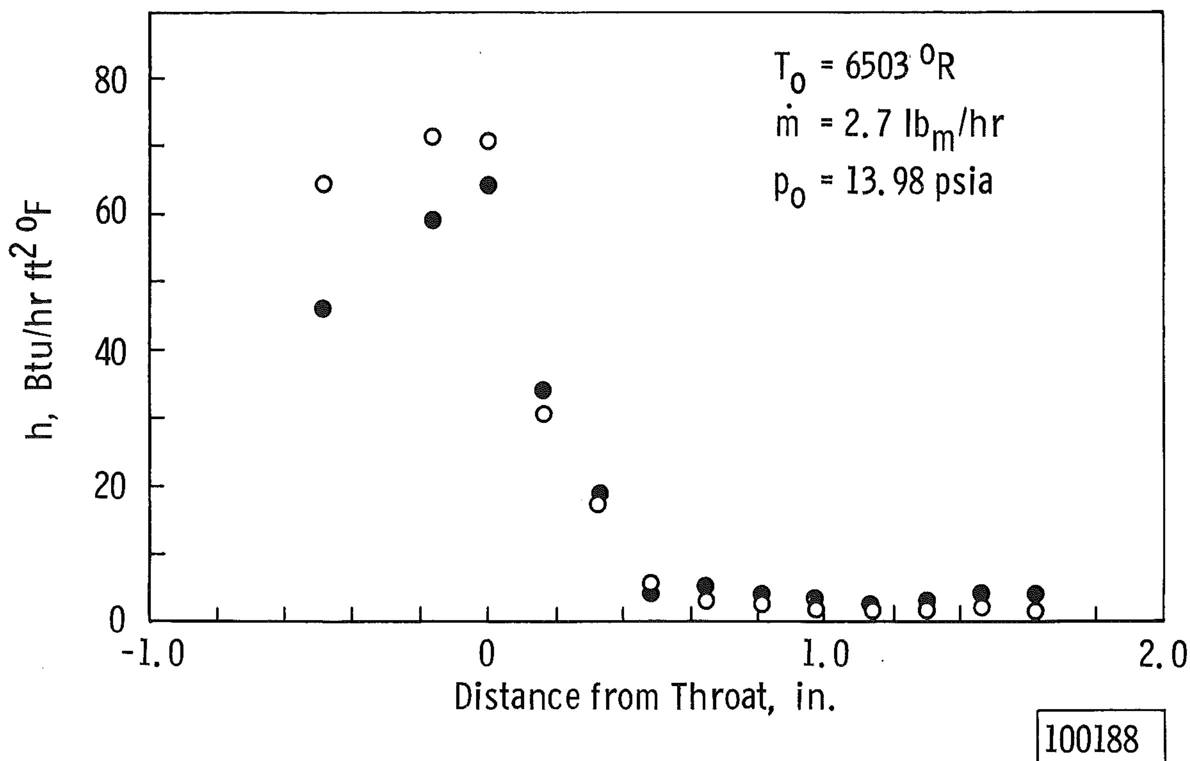


g. Run No. 7

Fig. 6 Continued



h. Run No. 8



i. Run No. 9

Fig. 6 Concluded

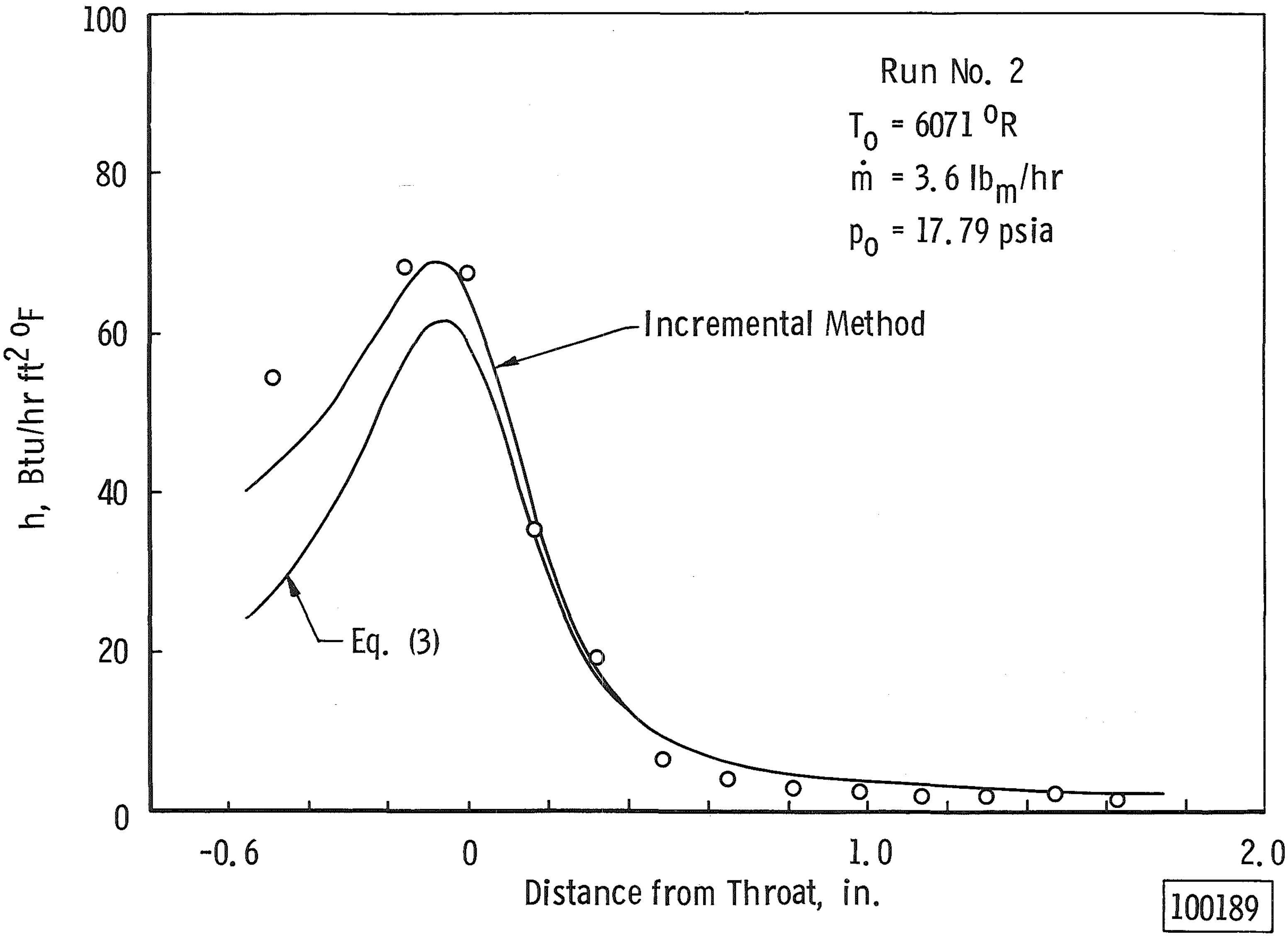


Fig. 7 Comparison of Simple Flat Plate and Incremental Flat Plate Methods

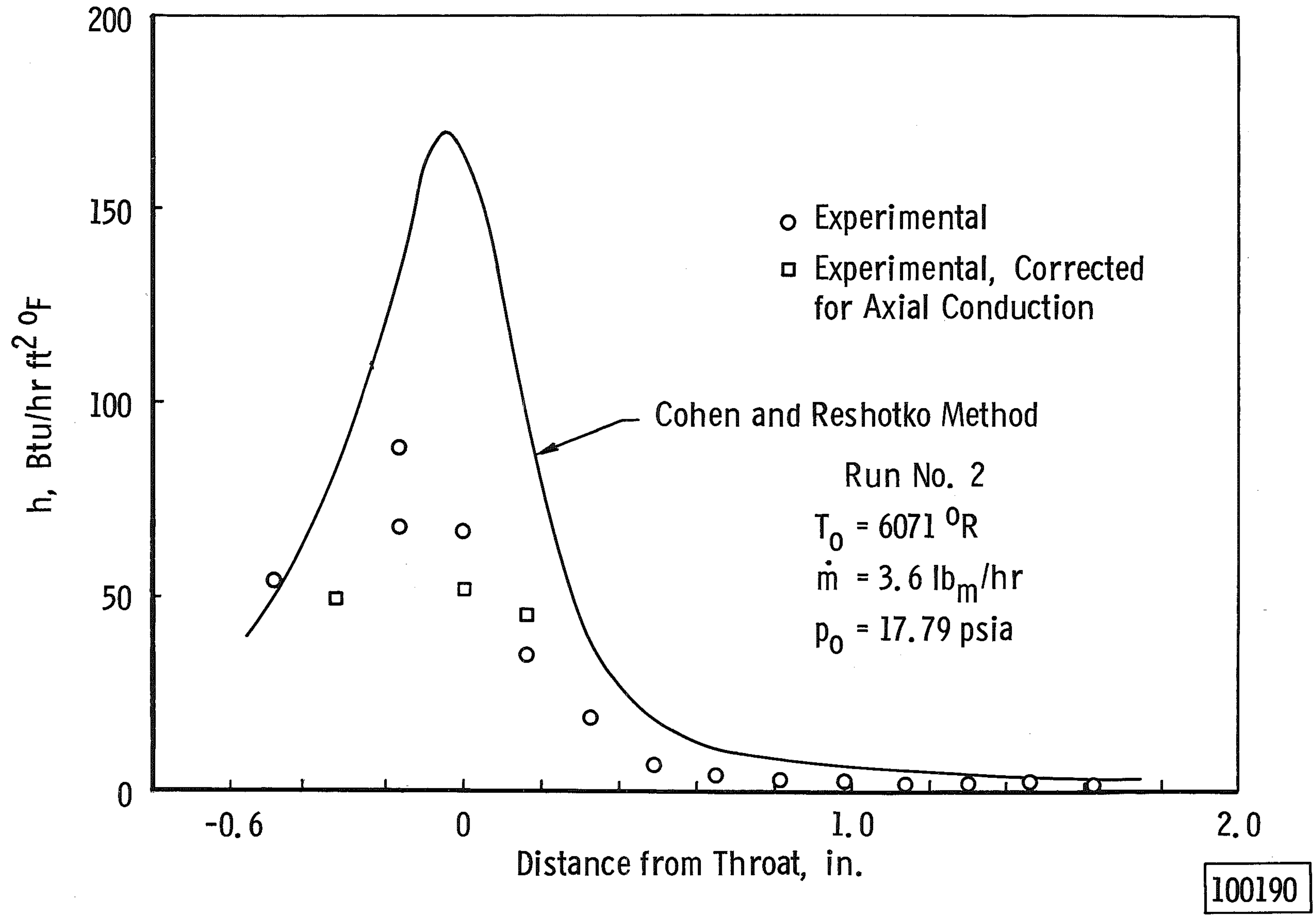


Fig. 8 Cohen and Reshotko Method Compared with Experimental Results

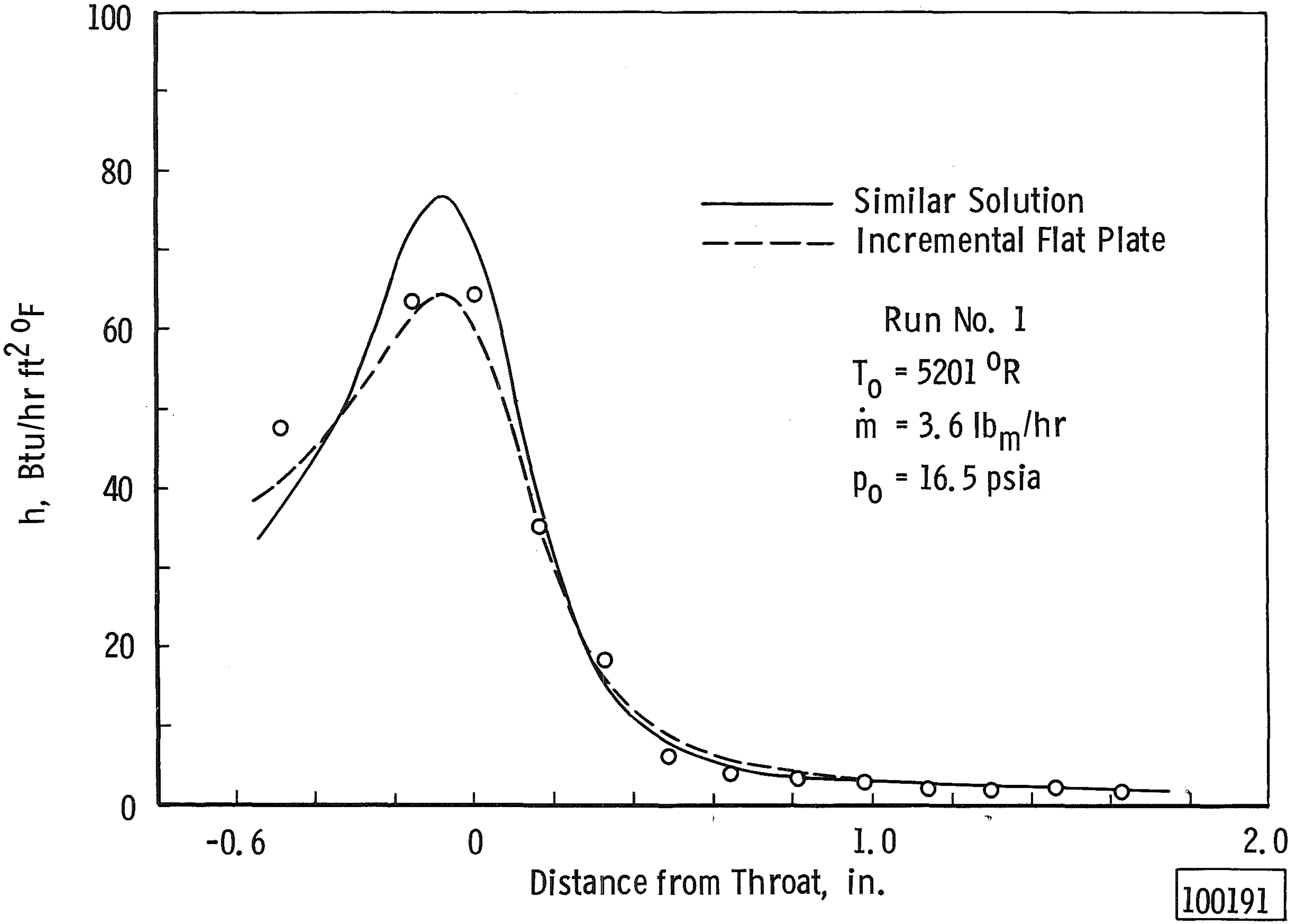


Fig. 9 Incremental Solution and Similar Solution for Run No. 1

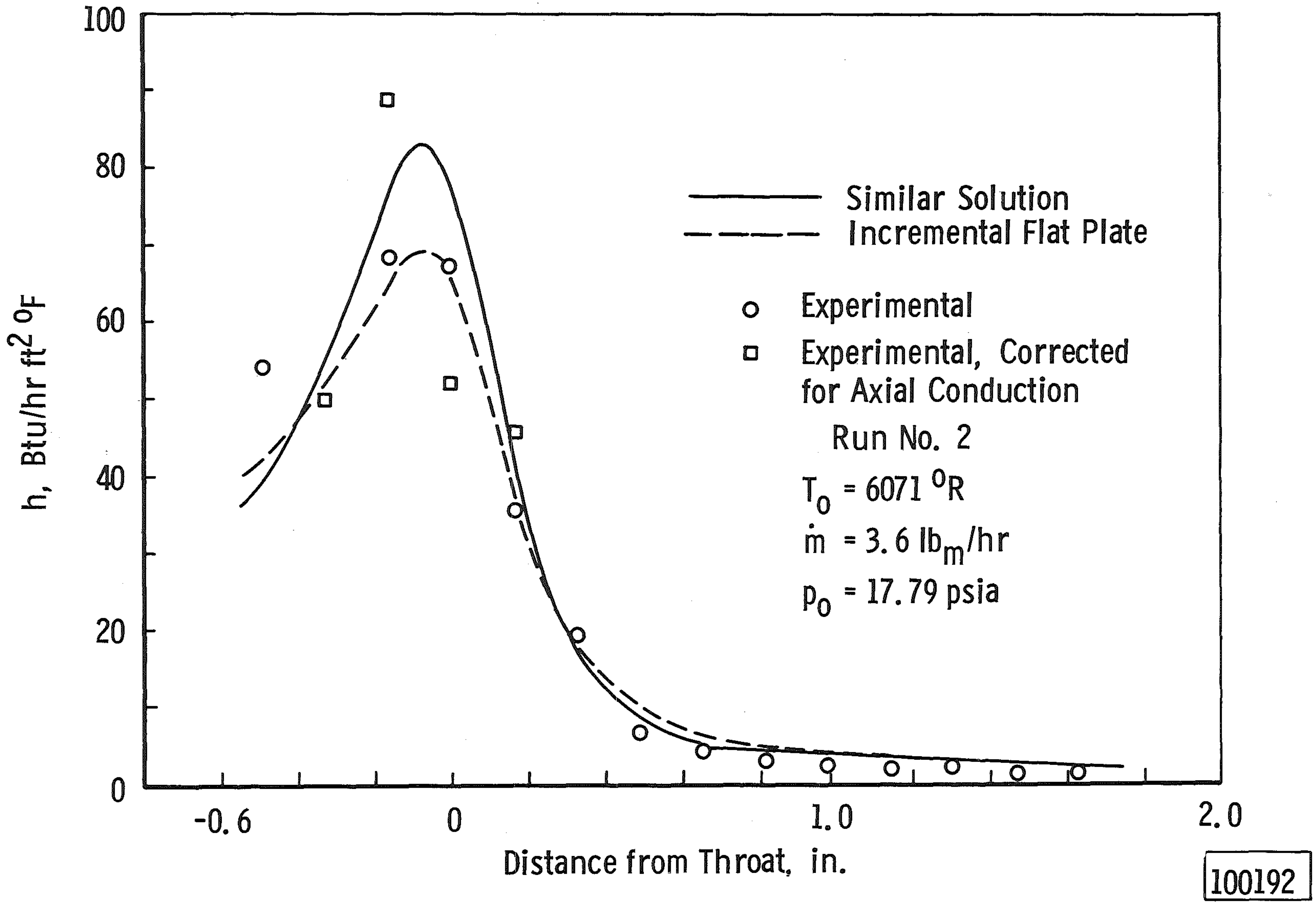


Fig. 10 Incremental Solution and Similar Solution for Run No. 2

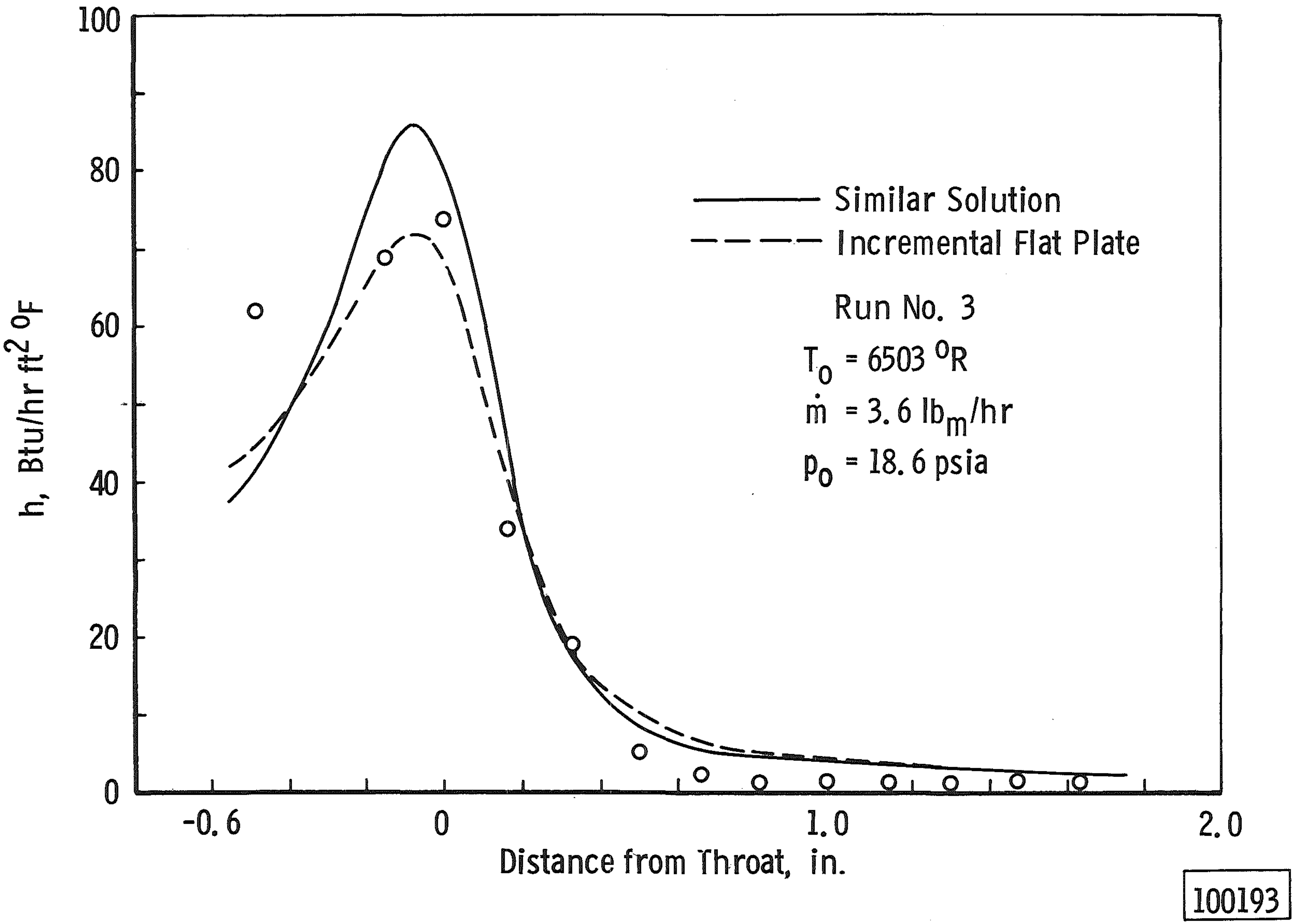


Fig. 11 Incremental Solution and Similar Solution for Run No. 3

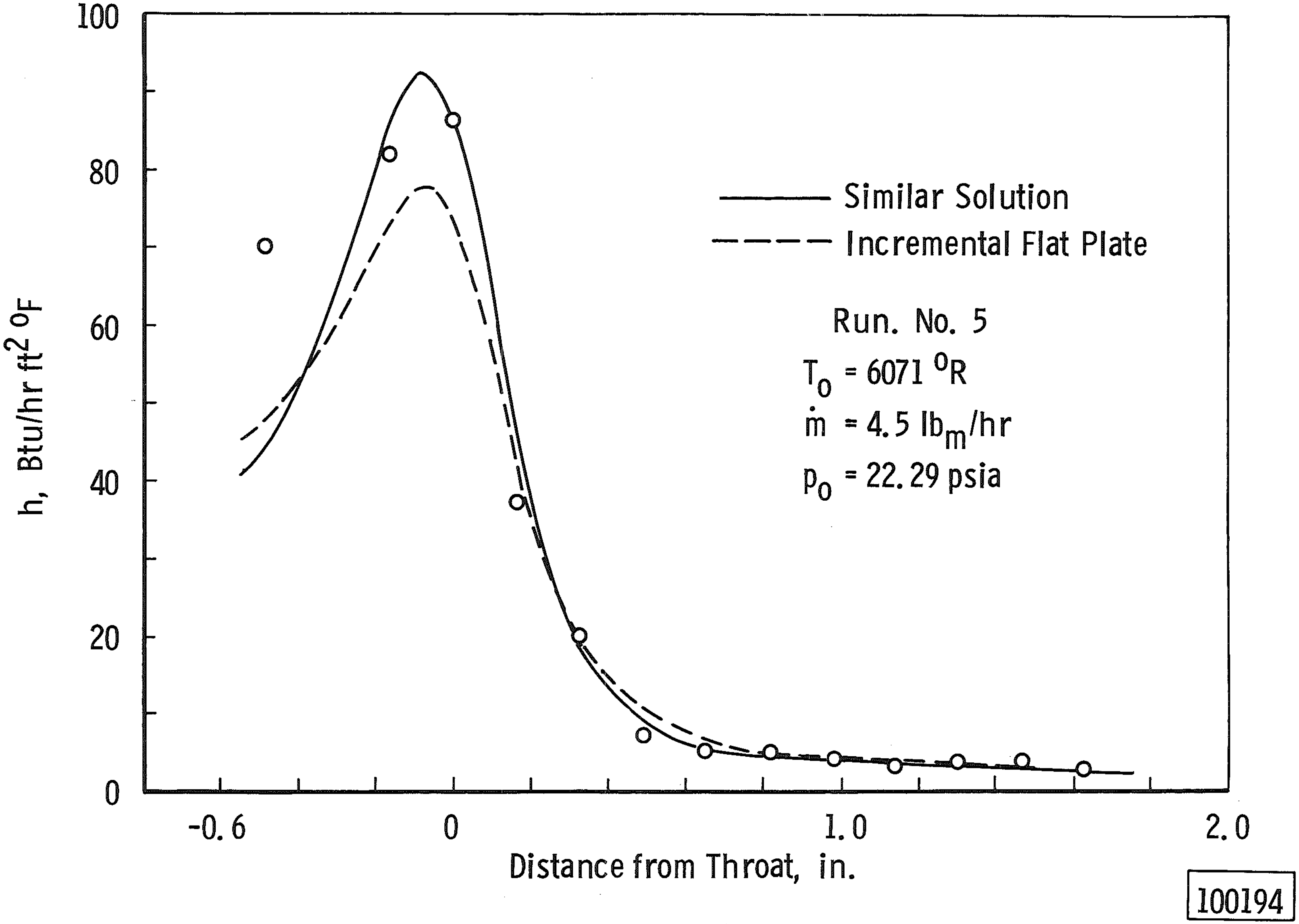
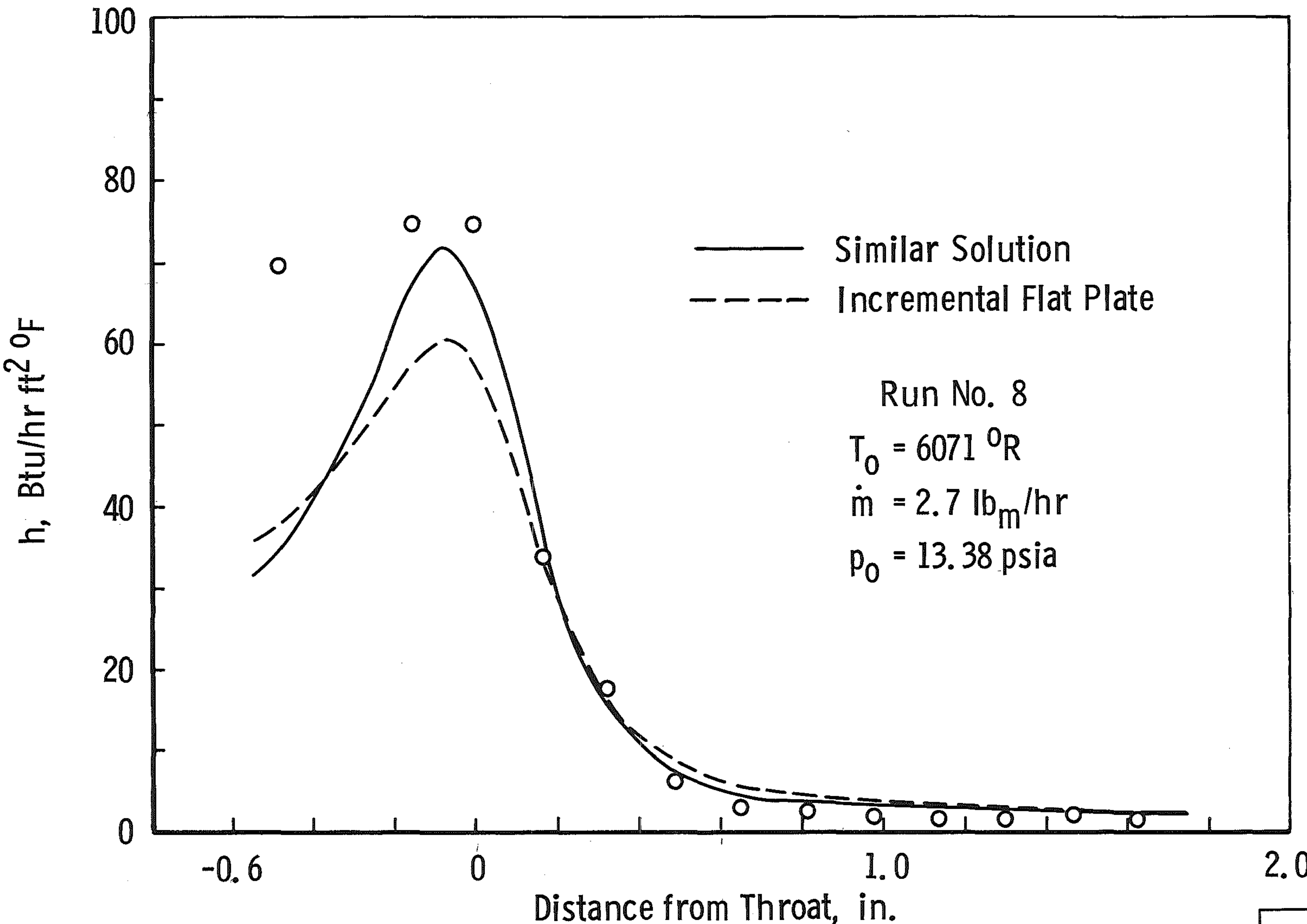


Fig. 12 Incremental Solution and Similar Solution for Run No. 5



100195

Fig. 13 Incremental Solution and Similar Solution for Run No. 8

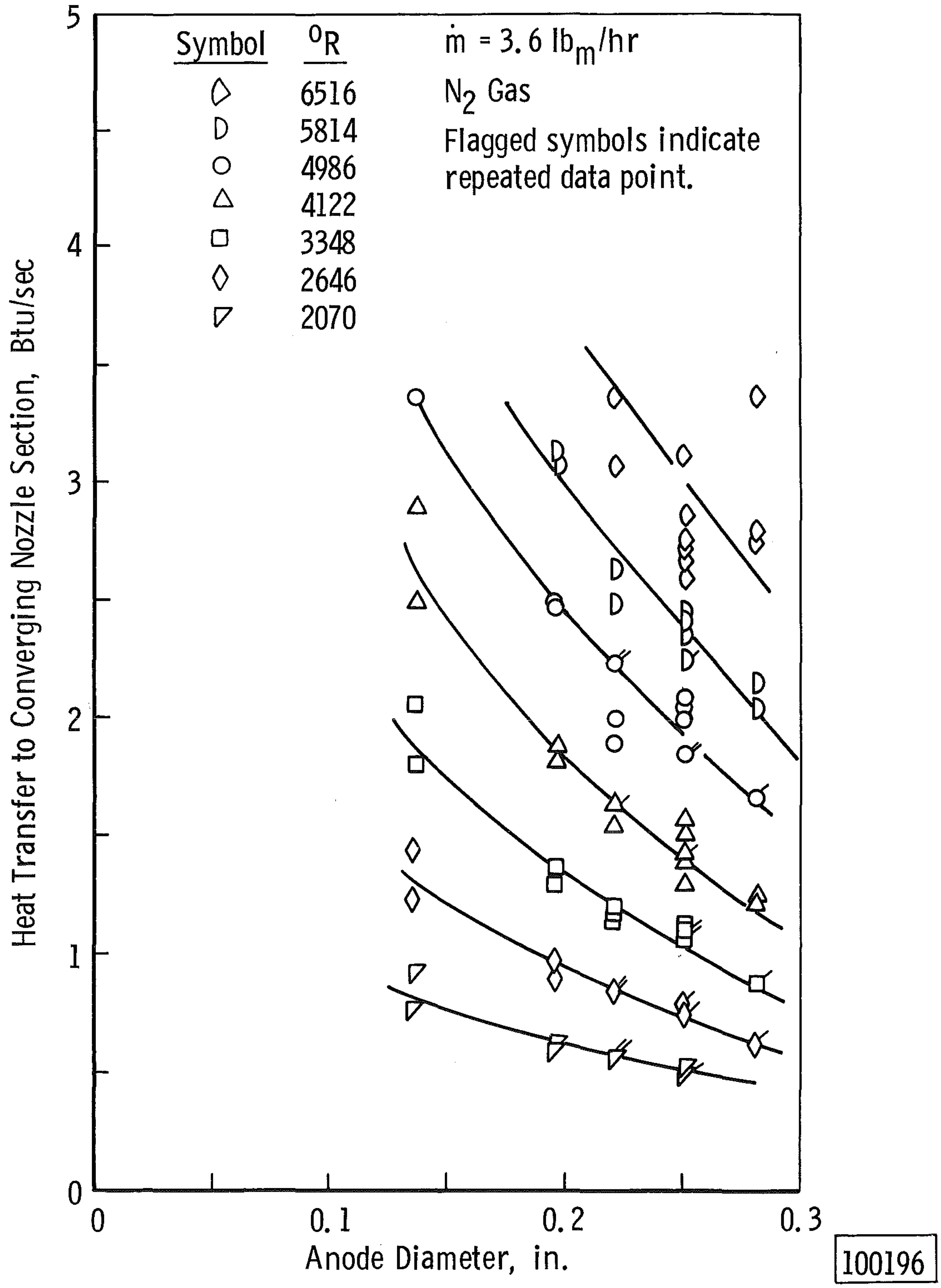


Fig. 14 Effect of Anode Diameter on Nozzle Convergent Section Heat Transfer